Methodology to determine the appropriate amount of excess air for the operation of a gas turbine in a wet environment

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1. Introduction

The thermal cycle of a power plant is affected by the conditions that are present at the place where it is installed, mainly ambient temperature, atmospheric pressure and the air’s relative-humidity. All these parameters have impact in the generated electric-power and the heat-rate during operation [1] and [2]. Among these variables, the ambient temperature causes the greatest performance variation during operation.

According to Felipe and Electo [3], while the air’s relative-humidity increases, the power generated by the combined-cycle plants also increases, provided the other parameters remain constant. In this case, the gas-turbine's efficiency is slightly reduced, as well as its power. However, the temperature of the gas-turbine’s exhaust-gases increases, and therefore the power generated by the steam cycle is increased. Nevertheless, the operation and performance of the thermal cycle depends on great measure of the place’s environmental conditions as well as the design condition [4]. Improvements in the design and operation of gas turbines have come along with advances in aerodynamics, thermodynamics and metallurgy. Such relationship has made possible for today’s gas turbines to withstand temperatures in the 1700 °C range, pressure ratios up to 34:1, internal compressor and turbine isentropic efficiencies up to 90 percent, and overall thermal efficiency up to 40 percent [5]. In the same way, the number of stages required in a compressor to achieve a specific pressure ratio, and consequently the overall gas turbine size, has been reduced allowing more compact and efficient gas turbines for a given power output.

However, the most important improvement that the gas turbine has experimented is, perhaps, an increase of the turbine gas inlet temperature. This has been possible thanks to recent developments on cooling techniques for turbine blades, and metallurgical advances [6]. The turbine inlet gas temperature is, of course, linked to exhaust temperature from the combustion chamber. It is a well known fact that the temperature during steady combustion in the combustion zone could greatly exceed the maximum allowable temperature by the turbine blades placed on the first stage of the turbine. In order to lower the temperature of the combustion gases to a tolerable level, an important amount of excess air is continuously supplied in order to keep the turbine inlet temperature at acceptable levels during steady state operation. To determine this amount of excess air, the humidity of the atmospheric air for the combustion...
needs to be taken into consideration. The effect of relative humidity on the operation of a gas turbine has been previously discussed by Usiyama [8]; there, the effect of ambient conditions, like temperature, pressure and humidity, over the gas turbine performance is analyzed. Usiyama’s work concludes that the effect of humidity must be considered in accurate calculations when the ambient temperature is above 30 °C, and the relative humidity is over 70 percent. The paper presented by Rice [9] concurs with the conclusions presented in [8]. Due to the importance of relative humidity in the determination of the amount of excess air required for the safe operation of a gas turbine, its role and effects are considered in this paper. Firstly, calculations considering dry air are presented and used as a point of reference, then the actual amount of excess air required for operation in a wet air environment is determined. It must be pointed out that previous works in the literature considering the effect of the relative humidity analyze only the effect over the plant’s global thermal efficiency, but do not consider the impact on the turbine inlet temperature, which is a point of major concern in this article. Previous works also disregard an analysis of how the thermal efficiency, but do not consider the impact on the turbine inlet temperature, which is a point of major concern in this article. Previous works also disregard an analysis of how the thermal efficiency and the work output depend on pressure ratio and excess air [10,7]. Bussman and Baukal [12] analyzed the effect of ambient conditions on the process heater efficiency.

2. System description

Fig. 1 shows a schematic diagram of a combustion chamber with its main components, combustion and cooling zones. Depending on turbine blade materials, and the availability and design of the turbine cooling systems, the combustion gas temperature at the turbine inlet ranges from 800 to 1700 °C. In order to attain these temperatures, excess air in the order of 600–100 percent needs to be supplied. However, large amounts of excess air cause the combustion in the turbine to become un-steady. That is why combustion chambers are divided into three sections, the combustion zone, and the mixture and dilution zone [11], as Fig. 1 shows. In the combustion zone, fuel is sprayed and stabilized by swirling vanes; primary air is mixed with fuel, making the combustion mixture approximately stoichiometric, which ignites the flame and keeps a steady temperature at approximately 2200 °C, depending on the type of utilized fuel. In the first stage of the mixture and dilution zone, secondary air is forced into the flame tube through small holes on the tube wall in order to achieve a complete combustion, and also to lower the temperature of the neighboring combustion zone. In the second stage of the mixture and dilution zone, tertiary air is supplied to further lower the temperature of the gases coming out from the combustion zone. The total amount of air that is delivered should be large enough to reach a permissible temperature value at the first stage of blades in the turbine.

3. Dry air composition

Atmospheric air is basically an Oxygen and Nitrogen mixture with slight quantities of Carbon Dioxide, Argon and Water Steam. Its composition slightly varies with humidity and altitude. When the presence of water steam is not considered in atmospheric air composition, the latter is known as dry air. This work considers the dry air composition as 21 percent Oxygen and 79 percent Nitrogen. Thus, the 79 percent N₂ fraction refers to the mixture of N₂, CO₂ and Ar, which is known as atmospheric Nitrogen. Therefore, it is assumed that one mol of dry air contains 0.21 mol of Oxygen, and 0.79 mol of Nitrogen. This is the typical composition of the atmospheric air that is utilized in the design, and analysis of internal combustion machines.

4. Wet air characterization

Wet air is simply a mixture of dry air and water steam. Here, let us assume that wet air behaves as an ideal gas, and also that the liquid phase embedded in the wet air mixture does not contain any dissolved gas. To determine the appropriate amount of excess air...
needed in the operation of a gas turbine, a thermodynamic analysis must take into consideration the ambient humidity.

The composition of wet air can be described in terms of the mol fractions of its constituents, dry air and water, in the following manner:

$$1 \text{ WA} = [\chi_{\text{DA}}] \text{ DA} + [\chi_{\text{H}_2\text{O}}] \text{ H}_2\text{O}$$  \hspace{1cm} (1)

where $\chi_{\text{DA}}$ and $\chi_{\text{H}_2\text{O}}$ are the dry air and water steam mol fractions present in the wet air mixture. These mol fractions are related with the atmospheric pressure $p_0$, the ambient temperature, through the saturation pressure of water $p_{\text{sat}}$, and the relative humidity $\phi$, as given in Eqs. (2) and (3):

$$\chi_{\text{H}_2\text{O}} = \frac{\phi \ p_{\text{sat}}}{p_0}$$ \hspace{1cm} (2)

$$\chi_{\text{DA}} = 1 - \frac{\phi \ p_{\text{sat}}}{p_0}$$ \hspace{1cm} (3)

To illustrate this matter, Fig. 2 presents the variation of the mol fraction of water in atmospheric air at 0.872 bar, in terms of the relative humidity for different temperature values. As the ambient temperature increases, the water fraction contained in atmospheric air increases for a given value of the relative humidity. At 80 percent relative humidity, the water fraction in atmospheric air increases from 1.2 to 7 percent in the temperature range that goes from 20 to 40 °C, as can be noted in Fig. 2.

The variability in the composition of atmospheric air, which is displayed in Fig. 2, affects the overall composition, and thermodynamic properties of the combustion mixture. Since properties like the specific heat of the mixture determine the final temperature of the combustion gases, the right amount of excess air for combustion should be properly assessed.

Likewise, Fig. 3 presents the water mol fraction in atmospheric air at 15 °C, in terms of the relative humidity for various atmospheric pressure levels. This figure shows that at 60 percent of relative humidity, the water fraction contained in atmospheric air increases from 1.4 to 2 percent when the atmospheric pressure decreases from 1.01 to 0.672 bar.

5. Minimum excess wet air for safe turbine operation

Conceptually, if a combustion process is supplied with an amount of air equal to the theoretic amount of air, the combustion could go to completion, and the product should contain neither fuel nor oxygen. In practice, this stoichiometric combustion process is not viable; the characteristic contact times involved in industrial combustion processes are not enough to achieve the complete stoichiometric reaction of molecules of fuel and oxygen. This is why in practice, in order to achieve the complete combustion of fuel, an amount of air, which exceeds the theoretically necessary amount of air is delivered for the oxidation of the hydrocarbon molecules of the fuel. Furthermore, the extra amount of excess air that needs to be delivered to the combustion process should also be large enough to bring the temperature of the combustion gases down to tolerable levels at the turbine inlet stage. A straightforward methodology to determine this minimum excess wet air for safe turbine operation is next presented.

5.1. Natural gas composition

Natural gas is a fuel extensively used in the operation of turbines and combined cycle plants. The composition of natural gas varies from deposit to deposit, and even from time to time during extraction. The natural gas volumetric composition utilized in this works is shown in Table 1.

The combustion reaction of hydrocarbons, $\text{C}_n\text{H}_m$, with atmospheric air as oxidizing element can be represented by the following set of relationships:

$$\sum_{i=1}^{k} [\chi_i] [\text{C}_n\text{H}_m] + [\chi_{\text{WA}}] \text{WA} \rightarrow [A] \text{CO}_2 + [N_{\text{WA}}/\chi_{\text{H}_2\text{O}} + B] \text{H}_2\text{O}$$

$$+ 0.21N_{\text{WA}}\chi_{\text{DA}} - \text{CO}_2 + [0.79N_{\text{WA}}/\chi_{\text{DA}}] \text{N}_2$$  \hspace{1cm} (4)

$$\text{WA} = x_{\text{DA}} 0.21\text{O}_2 + 0.79\text{N}_2 + [\chi_{\text{H}_2\text{O}}] \text{H}_2\text{O}$$  \hspace{1cm} (5)

$$A = \sum_{i=1}^{k} \chi_i n_i; \quad B = \sum_{i=1}^{k} \frac{\chi_i m_i}{2}; \quad C = \sum_{i=1}^{k} \chi_i \left( n_i + \frac{m_i}{4} \right)$$  \hspace{1cm} (6)

$$N_{\text{WA},l} = \left( 1 + \lambda \right) \frac{C}{0.21[\chi_{\text{DA}}]}$$  \hspace{1cm} (7)

Table 1

Natural gas volumetric composition.

<table>
<thead>
<tr>
<th>Natural gas composition</th>
<th>Volumetric Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methane</td>
<td>(CH₄)</td>
</tr>
<tr>
<td>Ethane</td>
<td>(C₂H₆)</td>
</tr>
<tr>
<td>Propane</td>
<td>(C₃H₈)</td>
</tr>
<tr>
<td>Butane</td>
<td>(C₄H₁₀)</td>
</tr>
<tr>
<td>Isobutane</td>
<td>(C₅H₁₀)</td>
</tr>
</tbody>
</table>

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Let us assume that $T_3$ is the temperature of the mixture of fuel and air after the compression process in the gas turbine, which is also the temperature at the inlet of the combustion chamber. Also, let us say that $T_3$ is a pre-established safe operational value for the turbine inlet temperature. For the case of known combustible composition, and adiabatic steady state combustion in the combustion chamber of the turbine, the energy balance associated with Eq. (4) is given by Eq. (8):

$$\sum_{p} N_p \bar{h}_i(T_2) = \sum_{R} N_R \bar{h}_j(T_3)$$  \hspace{1cm} (8)

By combining Eqs. (4) and (8), the necessary molar flow rate of excess wet air, $N_{WA}$, is determined:

$$N_{WA} = \frac{\bar{h}_f(T_f) - [A\bar{h}_{CO_2}(T_3) - B\bar{h}_{H_2O}(T_3) - C\bar{h}_{O_2}(T_3)]}{\bar{h}_{WA}(T_3) - \bar{h}_{WA}(T_2)}$$  \hspace{1cm} (9)

where

$$\bar{h}_f = \sum_{i=1}^{m} \left[ |C_i| \bar{h}_{in,i} \right] \left( \Delta h_f^p + \Delta h_i^f(T_f) \right) \sum_{i=1}^{m} |C_i| \bar{h}_{in,i}$$

$$\bar{h}_{H_2O} = (\Delta h_f^p + \Delta h_i^f(T_3))_{H_2O}$$

$$\bar{h}_{CO_2} = (\Delta h_f^p + \Delta h_i^f(T_3))_{CO_2}$$

Notice that the denominator in Eq. (9) represents the enthalpy change of wet air from the inlet of the combustion chamber, at $T_2$, to the inlet of the turbine, at $T_3$. The enthalpy of wet air can be computed at a given temperature $T$ with the following relationship:

$$\bar{h}_{WA}(T) = |x_{DA}| (0.21 \bar{h}_{O_2}(T) + 0.79 \bar{h}_{N_2}(T)) + |x_{H_2O}| \bar{h}_{H_2O}(T)$$  \hspace{1cm} (10)

Finally, substituting Eq. (9) in Eq. (7), and solving for the amount of excess air, $\lambda$, we obtain the desired result in Eq. (11):

$$\lambda = \left( \frac{N_{WA}}{N_{WAst}} - 1 \right) 100\%.$$  \hspace{1cm} (11)

Here, $N_{WAst}$ represents the stoichiometric quantity of wet air, which by definition corresponds to $\lambda = 0$, and can be computed directly from Eq. (7). In this manner, Eq. (11) provides a simple way to determine the minimum amount of excess wet air required for the safe operation of a gas turbine, with turbine inlet temperature equal to $T_3$.

On the other hand, for a given value of excess air, the corresponding turbine inlet temperature $T_3$ can also be determined by using an implicit iterative method in Eq. (11).

5.2. Thermal efficiency

The thermal efficiency, which depends on the pressure ratio, pressure losses, and maximum cycle temperature, can be expressed in a general form as follows:

$$\eta_{th} = \frac{y \eta_{isf}(1 - \frac{1}{\pi}) - \frac{1}{\eta_{isc}} (\pi^2 - 1) \frac{Z \pi^2}{\eta_{isc}} (\epsilon_1 + \epsilon_2)}{y - 1 - \frac{1}{\eta_{isc}} (\pi^2 - 1) \frac{Z \pi^2}{\eta_{isc}} (\epsilon_1 + \epsilon_2)}$$  \hspace{1cm} (12)

where $Z = R/C_{0}$ and $y = T_3/T_f$.

Since a reduction in the amount of excess air results in a higher turbine inlet temperature, accordingly with Eq. (12), at a given constant pressure ratio, the thermal efficiency of a gas turbine cycle improves with a reduced amount of excess air.

6. Results

As an illustrative example, consider the operation of a gas turbine cycle under the ambient conditions, and design specifications presented in Table 2. The volumetric composition of the natural gas utilized in the operation of the gas turbine is as given in Table 1.

The amount of excess wet air required to reach a specific turbine inlet temperature is determined from the relationships presented in Eq. (7) and Eq. (11). The amount of excess dry air is obtained likewise by making $x_{H_2O} = 1$ and $x_{DA} = 0$ in Eq. (4).

Fig. 4 presents the required amount of excess air, with either dry or wet air with 45 percent relative humidity, for different values of the turbine inlet temperature, $T_3$; the base case in Table 2 is represented by point B. The main tendency in the results show that the higher the tolerance in the value of the turbine inlet temperature, the lesser the amount of excess air that needs to be supplied for the operation of the gas turbine. Note also that the results corresponding to dry air provide a valid upper bound for the amount of excess air required for the operation of the turbine in a wet environment. Indeed, for the base case, a 4.96 percent reduction in the amount of excess air can be observed, with respect to the amount of excess air required for operation of the turbine in a dry environment. For a turbine inlet temperature of 1000 °C, points A and C in Fig. 4, the reduction in the amount of required excess air would be approximately 5.24 percent.

Fig. 5 presents the required amount of excess air for the operation of the gas turbine at different relative humidity levels, and several ambient temperature values. For a given ambient temperature, the results show that the required amount of excess air decreases with an increased relative humidity level; this outcome is linked to the larger heat capacity associated with mixtures of combustion gases with higher concentrations of water. In agreement with the results displayed in Fig. 2, the steepest slopes of the lines for higher ambient temperatures in Fig. 5, reveal the increasingly larger amounts of water that the atmospheric air can contain when the relative humidity rises.

Table 2

<table>
<thead>
<tr>
<th>Ambient conditions and design parameters for the operation of the gas turbine of the illustrative example.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>Ambient temperature, $T_0$</td>
</tr>
<tr>
<td>Ambient pressure, $p_0$</td>
</tr>
<tr>
<td>Relative humidity, $\phi$</td>
</tr>
<tr>
<td>Pressure ratio, $\pi$</td>
</tr>
<tr>
<td>Compressor efficiency, $\eta_{isc}$</td>
</tr>
<tr>
<td>Sum of relative pressure losses ($\epsilon_1 + \epsilon_2$)</td>
</tr>
<tr>
<td>Turbine inlet temperature, $T_{in}$</td>
</tr>
<tr>
<td>Turbine efficiency, $\eta_{turb}$</td>
</tr>
<tr>
<td>Power output, $P$</td>
</tr>
</tbody>
</table>

Fig. 4. Required amount of excess air versus turbine inlet temperature.
For the case of low relative humidity levels, Fig. 5 clearly shows that for the operation of the turbine at higher ambient temperatures, a larger amount of excess air is demanded in order to lower the temperature of the combustion gases to the design condition of 1243 °C. On the other hand, a reversal of this trend is observed at higher humidity levels, in which the operation of the turbine under a higher ambient temperature does not necessarily demand larger quantities of excess air for the operation of the turbine. Note, for instance, that below approximately 32 percent humidity level, a larger amount of excess air for the operation of the turbine is necessary at 50 °C ambient temperature than at 40 °C. The opposite is true above a 32 percent humidity level.

Fig. 6 shows that by increasing the compression ratio, a greater excess air is required in order to reach a given turbine inlet temperature. The last assumption is due to the increase of the compression ratio, the temperature at the end of the compression process increases; therefore it has a lower cooling effect at the dilution zone (Fig. 1) of the combustion chamber. Raising the compression ratio increases the excess air required to reach a given turbine inlet temperature.

According to Eq. (12), the Fig. 7 shows the net work output variation as a function of the cycle thermal efficiency at different excess air values (turbine inlet temperature), as well as to different pressure ratios. Thermal efficiency of gas turbine cycle increases as the excess air decreases (increasing turbine inlet temperature) keeping a constant compression ratio. However, it must be noted that for a given excess air (a given turbine inlet temperature), there is a pressure ratio which maximises thermal efficiency. Considering 600.29, 324.51 and 198.8 percent excess air, the turbine inlet temperatures are 800, 1000 and 1200 °C, respectively.

7. Conclusions

The amount of excess air that is supplied for the operation of a gas turbine influences the turbine inlet temperature, and the overall thermal efficiency of the thermodynamic cycle. Indeed, a workable reduction in the amount of supplied excess air can lead to a higher thermal efficiency value. A simple methodology has been presented in this paper to determine the value of the amount of excess air required for the operation of a gas turbine in a wet environment. The presented methodology, along with the illustrative example included in the paper, show that computations of excess air without the consideration of humidity can produce excess air values that exceed the actual need for excess air in the order of 5%. In this manner, this paper puts in evidence that dry environment calculations render an upper bound for the amount of excess air that is actually needed for the operation of the turbine in a wet environment. The presented results also show that, for a given ambient temperature, the required amount of excess air decreases with increased relative humidity levels, this due to the larger heat capacities that are associated with mixtures of gases with higher concentrations of water. As could be well expected, the operation of a gas turbine in low relative humidity environments demands higher amounts of excess air when operating with higher ambient temperatures. Nevertheless, the results in this paper show that this trend reverses when the turbine operates in a higher relative humidity environment.

An illustrative example

Stage 1

<table>
<thead>
<tr>
<th>Environmental conditions</th>
<th>Technological factors of compressor</th>
<th>Fuel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1 = 0.872$ bar</td>
<td>$\eta_{cc} = 0.86$</td>
<td>$M_f = 16.524$ $\text{kg/kg mol}$</td>
</tr>
<tr>
<td>$T_1 = 37.2$ °C</td>
<td>$\pi = 15.2$</td>
<td></td>
</tr>
<tr>
<td>$\phi = 45%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$[\text{H}_2\text{O}] = 0.965731$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$[\text{N}_2] = 0.03529$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{\text{dry}} = 28.4963$ $\text{kg/kg mol}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Stage 2

Compressor outlet pressure

$p_2 = \pi p_1 = 13.25$ bar

Compressor outlet temperature

Fig. 5. Required amount of excess air for different relative humidity levels and ambient temperatures.

Fig. 6. Excess air as function of the combustion gas temperature at the turbine inlet for different pressure ratio.

Fig. 7. Performance map showing the effect of pressure ratio and excess air (turbine inlet temperature) on a simple cycle.
Stage 3

Turbine inlet temperature: $T_3 = 1516.15$ K
The volumetric analysis of the products is $[\text{CO}_2] = 0.0354946$, $[\text{H}_2\text{O}] = 0.1013546$, $[\text{O}_2] = 0.1256477$, $[\text{N}_2] = 0.7375031$.
The molecular mass of products: $M_g = 28.0689289$ kg/kg mole where

$$
T_f(25 \text{ } ^\circ\text{C}) = -75244.95 \text{ kJ/kg mol}
$$

$$
T_{\text{CO}_2}(1243 \text{ } ^\circ\text{C}) = -393522 - 62719.86
= -330802.14 \text{ kJ/kg mol}
$$

$$
T_{\text{H}_2\text{O}}(1243 \text{ } ^\circ\text{C}) = -241826 - 48942.82
= -192883.18 \text{ kJ/kg mol}
$$

$$
T_{\text{O}_2}(1243 \text{ } ^\circ\text{C}) = 41191.86 \text{ kJ/kg mol}
$$

$N_{\text{WA,liq}} = 28.198 \text{ moles}_{\text{WA}}/\text{moles}_f$
The stoichiometric amount of moist air that enters the combustion chamber

$$
N_{\text{WA,liq}} = \frac{\sum_{i=1}^{k} |i| (n_i + \frac{m_i}{4})}{0.21 |\text{DPA}|} = 2.05175
$$

$$
N_{\text{WA,liq}} = 10.10042 \text{ moles}_{\text{WA}}/\text{mol}
$$
The amount of excess wet air required for the safe operation of a gas turbine, with turbine inlet temperature equal to $T_2 = 1243 \text{ } ^\circ\text{C}$

$$
\lambda = \left( \frac{N_{\text{WA,liq}}}{N_{\text{WA,liq}} - 1} \right) 100 = \left( \frac{28.198427}{10.10042} - 1 \right) 100 = 178.48\%
$$

The fuel-air ratio for this combustion process is

$$
\text{RCA}_{\text{WA}} = \frac{0.21 |\text{DPA}| M_f}{(1 + \lambda) \sum_{i=1}^{k} |i| (n_i + \frac{m_i}{4}) M_{\text{WA}}}
$$

$$
= \frac{0.21 |0.96731| 16.524}{(1.0 + 1.7848) 10.05175} 28.4963 = 0.020615469 \text{ kg/g}_{\text{WA}}
$$

The specific heat at constant pressure and the enthalpy of products, with turbine inlet temperature equal to $T_3 = 1243 \text{ } ^\circ\text{C}$

$$
h_3 = [\text{CO}_2] h_{\text{CO}_2}(T_3) + [\text{H}_2\text{O}] h_{\text{H}_2\text{O}}(T_3) + [\text{O}_2] h_{\text{O}_2}(T_3) + [\text{N}_2] h_{\text{N}_2}(T_3) = 1460.5356 \text{ kJ/kg}
$$

Turbine inlet pressure

$$
p_3 = p_2 [1 - \Delta \rho_{cc}] = 13.25 [1 - 0.023] = 12.9495 \text{ bar}
$$

Stage 4

Turbine outlet pressure

$$
p_4 = p_1 [1 + \Delta p_{tc}] = 0.872 [1 + 0.03326] = 0.9010027 \text{ bar}
$$

Turbine isentropic efficiency, $\eta_{\text{is}} = 0.90$

Turbine outlet pressure

$$
T_4 = T_3 \left( \frac{p_4}{p_3} \right) = 885.73 \text{ K}
$$

The specific heat at constant pressure and the enthalpy of products, with turbine outlet temperature equal to $T_4 = 612.5824 \text{ } ^\circ\text{C}$:

$$
C_p = [\text{CO}_2] C_{\text{PCO}_2}(T_4) + [\text{H}_2\text{O}] C_{\text{PH}_2\text{O}}(T_4) + [\text{O}_2] C_{\text{PO}_2}(T_4) + [\text{N}_2] C_{\text{PN}_2}(T_4) = 1.203853 \text{ kJ/kg K}
$$

$$
h_4 = [\text{CO}_2] h_{\text{CO}_2}(T_4) + [\text{H}_2\text{O}] h_{\text{H}_2\text{O}}(T_4) + [\text{O}_2] h_{\text{O}_2}(T_4) + [\text{N}_2] h_{\text{N}_2}(T_4) = 669.350756 \text{ kJ/kg}
$$

The net specific work output of the turbine, i.e. power output turbine per unit air mass flow, is

$$
W_m = (h_3 - h_4) - (h_2 - h_1) = 340.8741 \text{ kJ/kg}
$$

Heat supplied to the working fluid

$$
q_A = (h_3 - h_2) = 997.706743 \text{ kJ/kg}
$$

Thermal efficiency

$$
\eta_{\text{th}} = \frac{W_m}{q_A} = 0.34166
$$

References